The solution of a class of two-parameter eigenvalue problems and a connection with the implicitly restarted Arnoldi method

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The stability of the steady state of a nonlinear dynamical system can often be determined from a so-called linear stability analysis: the nonlinear system is linearized around the steady state. The eigenvalues of the Jacobian determine the stability of the solution: the solution is stable iff all eigenvalues lie in the left-half plane. For checking the stability of large scale systems, it is sufficient to verify whether the right-most eigenvalues of the Jacobian lie in the left half plane. Dynamical systems usually have a physical parameter that may have a large impact on the solution. Solutions may become unstable. At a Hopf point, the Jacobian matrix has a pair of purely imaginary eigenvalues where the other eigenvalues lie in the left half plane. The determination of Hopf points is not always straightforward. One idea is to monitor the right-most eigenvalue when the parameters change, and use a continuation method to check where the eigenvalue crosses the imaginary axis. However, such methods may not always be reliable.

We will discuss a new method that directly computes the values of the parameter whose Jacobian matrix has a pair of purely imaginary eigenvalues, rather than monitoring the right-most eigenvalues. The problem is formulated as an inexact inverse iteration method that requires the solution of a sequence of Lyapunov equations with low rank right hand sides. It is this last fact that makes the method feasible for large systems. The power of our method is tested on numerical examples. We will show a connection with the implicitly restarted Arnoldi method.

References

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